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Field measurements and modeling of dilution in the wake of a US navy frigate

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Abstract

A field measurement and computer modeling effort was made to assess the dilution field of pulped waste materials discharged into the wake of a US Navy frigate. Pulped paper and fluorescein dye were discharged from the frigate's pulper at known rates. The subsequent particle and dye concentration field was then measured throughout the wake by a following vessel using multiple independent measures. Minimum dilution of the pulped paper reached 3.2×10^5 within 1900 m behind the frigate, or about 8 min after discharge. Independent measures typically agreed within 25% of one another and within 20% of model predictions. Minimum dilution of dye reached 2.3×10^5 at a down-wake distance of approximately 3500 m, or roughly 15 min. Comparison to model measurements were again within 20%. The field test was not only successful at characterizing wake dilution under one set of at-sea conditions, but was successful at validating the computer model used for assessing a wide range of ships and conditions.

Keywords: Shipboard discharges; Wake dispersion; Marine pollution; Mathematical models; Waste disposal

1. Introduction

This report describes a field measurement and computer-modeling effort designed to characterize the near-field exposure of pulped paper wastes discharged into the wake of an US Navy frigate. Assessment of the potential environmental impact of these wastes to marine organisms and its eventual fate requires accurate estimates of the exposure field, in both time and space. To provide these estimates for a variety of vessel types and operating conditions, a numerical wake-mixing model (TBWAKE) originally developed to assess wake bubbles, was modified to handle the characteristics of the solid and liquid waste discharge materials. The full-scale field test provided a way to validate the model predictions. The specific objectives of the full-scale field study were to:

- Measure at various downstream locations and depths the cross-wake concentration of a passive scalar, fluorescein dye, discharged into the wake of a frigate moving at 8 and 15 kts. Estimate minimum and average cross-wake dye dilution levels as a surrogate of the liquid phase of the waste stream.
- Measure at various downstream locations and depths, the average cross-wake concentration of pulped paper and fluorescein dye, discharged into the wake of a frigate moving at 8 kts. Estimate average pulped paper dilution levels, the solid phase of the waste stream.
- Compare dilution results with computational model (TBWAKE) simulations to assess the model's capability to predict dilution levels under varying ship and sea conditions.

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2. Technical approach

The basic test concept was for the USS *Vandergrift*, henceforth referred to as the frigate, to operate its solid waste pulper under controlled conditions while the measurement platform, the RV *Acoustic Explorer*, henceforth referred to as the AX, maneuvered behind making in situ and flow-through seawater measurements. A specific type of white photocopy paper and fluorescein dye were discharged from the frigate's pulper at specified rates. The dye and paper were both measured using fluorescent measurement techniques while the paper particles were also measured gravimetrically. The specific photocopy paper employed had an inherent fluorescent intensity that was used for particle quantification.

While the frigate was discharging paper and dye along a straight track, the AX transited in the wake in two geometries. In the first geometry termed "Follow-the-Leader" (FTL), the AX held position with its measurement equipment centered in the wake at three discrete distances behind the frigate: 5, 10, and 15 frigate ship-lengths. These measurements were designed to identify the average concentration of dye and paper particles at the specified distances back. In the second geometry termed "Serpentine" (SERP), the AX started close in behind the frigate, then fell back over time as a result of criss-crossing the full lateral extent of the wake. These measurements were designed to look at maximum concentrations of dye and the cross-wake distribution as a function of distance back.

The tests were run under as close to standard operating conditions as possible given ship and measurement limitations. An exception to this was that all but one set of measurements were made while the frigate discharged paper at a nominal rate that was about 2.5 times normal (110 $\mathrm{kg\,h^{-1}}$) to enhance the ability to detect paper particles. On one occasion, the rate was lowered to the nominal operational discharge rate of 45 $\mathrm{kg\,h^{-1}}$. Measurements were made while the frigate was transiting at either 8 or 15 kts.

3. Experimental methods

3.1. Test overview

The field test was conducted 8–24 km off the coast of San Diego, California, during the period 8 through 16 February 1996. Water depths in the test area were at least 200 m with a surface mixed layer of about 20 m. Excellent weather and sea-state 1 conditions prevailed throughout the test period. Daily operations began by performing standard hydrographic measurements aboard the *AX* to determine the depth of the mixed layer and to assess background water clarity using a light transmissometer. Once these measurements indicated appropriate field conditions, the full overboard sampling system described below was deployed and spun up to full operational status. Background particulate load and dye measurements were then made prior to rendezvousing with the frigate. Upon arrival of the frigate, the two ships positioned themselves appropriately in one of the two geometries and the tests began. The ships then traveled randomly in more or less straight lines within an operations area that was about 500 $\mathrm{km^2}$. Ship directions were altered when occasional patches of plankton, observed in plankton net samples (described below), were encountered. Multiple dye and pulp samples were collected under each test condition, and most test conditions were repeated at least twice.

3.2. Discharge platform configuration—frigate

The frigate has, at waterline, a length, beam, and draft of 125.9, 13.7, and 6.7 m respectively. It has a displacement of about 3000 t and is propelled by a single clockwise rotating screw. Its on-board "small" pulper was installed a few months prior to the field tests. The "small" refers to its physical size as well as its discharge rate that is nominally 45 $\mathrm{kg\,h^{-1}}$. A differential global positioning system (DGPS) was placed onboard the frigate to track its position during the tests.

Concentrated (40%) liquid fluorescein dye (Keystone Aniline Corporation, C.I. acid yellow 73) was delivered into the input side of the pulper using a peristaltic pump. The delivery rate of 0.8 $\mathrm{l\,min^{-1}}$ was monitored using an in-line flow meter. White photocopy paper (Hammerhill 24-lb Premium Business Laser Paper) was delivered manually into the pulper. The input rate was determined by timing the number of reams placed into the input side of the unit at a nominal feed rate of 45 or 110 $\mathrm{kg\,h^{-1}}$. The pulped paper/dye stream was then merged with the ship's seawater firemain, as per standard operating procedures, before being discharged amidships on the starboard side of the ship at a rate of 190 $\mathrm{l\,min^{-1}}$. The resultant mixture at the point of discharge had a dye concentration of 2 $\mathrm{g\,l^{-1}}$, equivalent to 6.7 $\mathrm{g\,s^{-1}}$ (density = 1.25 $\mathrm{g\,ml^{-1}}$), and pulped paper concentration of either 3.9 or 9.6 $\mathrm{g\,l^{-1}}$. Samples of the effluent just before it went overboard were randomly collected (and stored frozen) for confirmation.

The pulper was run for several minutes with dye and paper to stabilize the discharge rate prior to the start of each test. Once the discharge rate was stable, the ships positioned themselves appropriately and the measurements began. During FTL the runs, dye was continuously discharged only during the first 10 min. Thereafter, the dye was then pulsed to mark the wake centerline.

3.3. Measurement platform configuration—AX

A suite of in situ and onboard measurement devices was installed aboard the AX to measure particle and dye concentrations. A towed array was assembled which consisted of a high-volume seawater pumping system to provide samples for pulped paper analysis and for flow-through dye measurements. The array also consisted of a chain of in-fairing fluorometers to provide in situ dye measurements. The array was towed 6 m off the starboard beam using the ship's service crane, which was a sufficient distance from the ship so that the data collection was not influenced by the AX's own wake. Onboard, computers were used to acquire data from the in situ array, the flow-through dye fluorometers, as well as from the ship's DGPS navigation system. Additionally, the ship's radar was used to monitor ship separation distances.

3.4. Pulped paper particle collection

The high-volume seawater-pumping system used to collect pulped paper particles consisted of three 2.5 cm internal diameter reinforced plastic hoses taped to a 5-mm hydrowire holding a 225 kg dead weight. The lengths of the three tubes were such that water was sampled from depths of about 1.0, 4.6, and 9.5 m when the system was towed at 8 kts during FTL runs (sampling was slightly deeper when doing SERP runs). Pressure transducers installed at the end of each tube were used to monitor the intake depth.

Topside, the three hoses were plumbed into self-priming pumps that pumped seawater to a "tee" assembly at approximately $0.75\ \mathrm{l\,s^{-1}}$. At the "tee" 80% of the water flowing up each hose was delivered to a net manifold while 20% was delivered to a flow-through fluorometer. The net manifold fed three 1-m long, 20-μm mesh plankton net stockings. Each net was dedicated to one of the three sample hoses. At the bottom of each net was a PVC cod end where particles were collected, typically over a 10-min period. The cod ends were switched in a manner to maintain continuous sample collection during each test. The pulped material accumulated in the cod ends was processed immediately after collection.

3.5. Pulped paper particle concentration analysis

The mass of pulped paper collected from the high-volume filtration system was determined with two independent methods, fluorometrically and gravimetrically, as described below. The net-filtered samples were blended then split for each analysis. Concentration of the material was then determined by dividing the total mass collected by the total seawater volume filtered, accounting for filtering efficiency and correcting for background fluorescence or particulate load.

The fluorescent nature of the optical bleaching agents in the pulped white paper was used for sample mass determinations. A Turner Designs Model 10-000R fluorometer was used to determine the mass of pulped paper by measuring its fluorescence using an excitation waveband of 310–390 nm and emission waveband of 420–490 nm. At these wavebands, there was no interference with the fluorescein dye. The fluorometer was calibrated using serially diluted mixtures of a stock pulp solution containing a known concentration of paper. The resultant mass versus fluorescence calibration was used to quantify the mass of the test samples.

Mass of the samples was also determined by filtering each sample through a pre-weighed glass fiber filter (0.4 μm), drying, then weighing it on an electronic balance.

Filtering efficiency of the net manifold system was estimated by comparing the mass of pulped paper collected directly from the pulper (into a 20-l bucket) with those collected after pumping the material through the net filtration system. The filtering efficiency averaged 80% as measured by both fluorescent and gravimetric analysis.

3.6. Fluorescein dye concentration analysis

Two independent measures of fluorescein dye concentration were made during the field tests. The primary measurements were made using a chain of five in situ fluorometers deployed as part of the towed array. The second set of measurements were made on seawater flowing from the high-volume pumping system hoses through three separate onboard fluorometers. The in situ measurements provide the highest spatial and temporal resolution while the flow-through measurements provided more of an integrated measurement technique because of mixing within the hose system. Comparison data from the two measurement techniques were made after accounting for hose lag times and sample depth differences.

The towed fluorometer chain system consisted of five in-fairing fluorometers developed at JHU/APL. Sample depths of the individual sensors were monitored with pressure transducers. For the FTL geometry, sample depths averaged: 1.0, 2.0, 4.5, 7.7, and 9.0 m. Sample depths during the SERP geometry averaged: 2.7, 4.5, 6.5, 9.5, and 11.5 m.

Seawater passing through the fluorometer was optically excited near 490 nm and its fluorescence emission detected near 525 nm. The fluorometers were calibrated before and after the survey by placing them into a stock solution of fluorescein dye and serially diluting it, noting the fluorescence voltage at each known dye concentration. The calibration was performed over a range of dye concentrations between 0.3 and $300\ \mathrm{\mu g\,l^{-1}}$. The log–log linear regression of the fluorescence voltage as a function of concentration was used to calculate sample concentrations.

Water flowing up from the seawater pumping system was partially diverted through three flow-through Turner Model 10 AU fluorometers, one for each depth. Sample depths for the FTL runs averaged 1.0, 4.6, and 9.5 m. During SERP runs the sampling depths averaged 1.4, 5.6, and 10.3 m. The seawater passing through the fluorometer flow cell was optically excited at 455–510 nm and resultant fluorescence emission measured between 510 and 700 nm. Each of the flow-through fluorometers was calibrated before and after the surveys. This was done by recirculating seawater in the flow-through system and making serial additions of a primary fluorescein standard (0.026%), made from the 40% fluorescein dye used in the field tests. The resultant fluorescence signal in volts was recorded at each addition spanning a concentration range of 3–2700 $\mu g\,l^{-1}$. Dye concentrations of seawater measured during the field test was determined using a polynomial regression of the fluorescence signal in volts as a function of dye concentration.

4. Data processing

4.1. Data acquisition

Two real-time data acquisition computers were used to record navigation, fluorometer, and sample depth data. The in situ fluorometer and depth data from the towed array were displayed and recorded at 4 Hz along with GPS time on one computer. The flow-through fluorometers, pressure transducers, and DGPS data were displayed and recorded at 1 Hz on a second data acquisition system. All measurements were time matched to co-locate all data spatially. While the computers were not used to record pulped paper sample data or radar records, these data were logged manually using the GPS time recorded on the computers.

Because the DGPS aboard the frigate was inoperable during the serpentine tests, the distance between the frigate and *AX* was determined from the radar record. Discrete radar measurements of distance were regressed as a function of time. The resulting regression line was used to calculate, at any time during the test, the relative locations of the frigate and *AX*. The surface location of the towed array was estimated visually and added to the ship separation data to estimate the distance between measurement location and frigate.

4.2. Data visualization

Visualizations of the dye concentration data were generated using Golden Graphics Surfer® software. Using the true relative surface positions of the two ships, the calibrated dye data for each depth were gridded and interpolated to produce both plan and cross-sectional views of the dye for each of the SERP runs. Data from the plan views were interpolated on approximate grid size of $100x$ by $40y$ using an octant search radius of $1000x$ and $10y$, and a correlation length scale of $1000x$ and $100y$. Cross-sectional plots were produced for specific distances behind the frigate using the depth dependent data from each sensor. These data were interpolated on an approximate grid size of $100y$ by $40z$ using an octant search radius of $10y$ and $7z$, and a correlation length scale of $20y$ and $10z$. A threshold of 1 $\mu g\,l^{-1}$ was used as a cutoff value for display and statistical calculations, a value that was roughly three standard deviations above background.

5. Sampling summary

5.1. Pulped paper data

Pulped paper samples were collected from three depths (1.3, 4.6, and 8.8 m) on a total of seven FTL runs at 5, 10, and 15 ship-lengths downstream from the frigate. Samples were also collected at one ship-length back on two SERP runs. The FTL runs were repeated three times at the 5 ship-length separation distance (once at the lower discharge rate), and twice at 10 and 15 ship-length distances. During each run, two to seven replicate net-filtered sample sets were collected, depending on the length of the run. Background net samples were collected before and/or after each of the FTL runs (19 sets). A total of 207 samples from 69 sample sets (three depths) were collected and measured both fluorometrically and gravimetrically. An additional 13 samples obtained directly from the frigate's pulper discharge were analyzed gravimetrically.

5.2. Fluorescein dye data

Fluorescein dye measurements were made on the seven FTL runs. The data during these runs were analyzed quantitatively for the flow-through fluorometers only. Dilution calculations were calculated over a 10-min time period during each run when the dye was continuously pumped. Dye measurements made at all other times on the FTL runs were used only to qualitatively assess how well the centerline of the wake was tracked during the test. A total of six serpentine runs, three each at frigate speeds of 8 and 15 kts, were also made. During these runs, the wake was crossed 12–29 times. Fluorescein dye measurements were made continuously on these runs using both the in situ fluorometer array (primary method) and the flow-through fluorometers.

6. Computational model

The computational model TBWAKE, developed to study ship microbubble wakes (Smith and Hyman, 1987; Hyman et al., 1987; Nguyen and Hyman, 1988a,b), was modified to simulate the near field (<30 ship-lengths) dispersion of particles and liquid discharged into the wake of US Navy ships. The original model had previously been partially verified with full-scale experiments on many classes of US Naval surface craft under a wide range of oceanic conditions (Hyman, 1990, 1994).

The near field numerical simulation of wake dispersion is based on standard convective–diffusive modeling of continuous fluid fields. The model uses a parabolic Navier–Stokes solver in which the equations describing the instantaneous flow field were ensemble averaged. The result is a set of three-dimensional differential equations that are elliptic in character and solved assuming that the streamwise pressure and stress gradients are negligible, and that there is no flow reversal. Because of the assumptions used, the simulations cannot be initiated until after about a half ship-length (60 m). At this distance, all dependent variables are specified, forming what is termed the initial data plane (IDP). The numerical algorithms then propagate those data downstream. Estimation of the IDP is the weakest link in the simulation process but should provide better than order of magnitude dilution estimates. A series of runs with different locations of the initial concentration field within the IDP showed little effect on the concentration distribution at 3000 m behind the ship.

The original model algorithms employed a surface boundary condition of the zero gradient type. While a gradient-free boundary condition is appropriate for the passive scalar case, it is not suitable for the particle case, resulting in particles being introduced at the ocean surface. Consequently, a zero flux condition was imposed at the surface to first order accuracy. The outcome of this approximation was a 10–25% cumulative introduction of mass over the first 3000 m of the ship's wake.

The model discharge was considered to be composed of particles that are sufficiently numerous such that they can be treated as a continuum in both space and size (diameter). The model used the range of particle sizes from 100 to 3000 μm, a density of 1.54 $g\,cm^{-3}$, and settling velocities described in Chadwick et al. (1996). A continuous particle population (with units # $m^{-3}\,\mu^{-1}$), proportional to r^{-3}, was defined and discretized as shown in Table 1. Particles were not allowed to change size (coalesce, shear, or decompose), and they were presumed to be always falling at their (size dependent) terminal velocity.

Simulations were run for both 8- and 15-kts frigate speeds for both unstratified and stratified ocean conditions. The computational domains varied with ship speed but generally ranged laterally to about 100 m, vertically to about 30 m, downstream from the IDP to about 4000 m. The $200 \times 91 \times 41$ (streamwise by horizontal by depth) grids had a high density near the surface and near the IDP but were uniform in the lateral direction. The total amount of mass flux through the IDP was determined by the rate of discharge from the ship, 6.7 $g\,s^{-1}$ of dye and either 3.9 or 9.6 $g\,l^{-1}$ pulped paper. Particle concentrations were computed by summing all particle sizes. The passive scalar simulations treated the discharge as if it was a neutrally buoyant dye. All simulations used background diffusivity of 10 $cm^2\,s^{-1}$.

Table 1
Pulped paper characteristics used as model inputs

Radius (μm)	Terminal velocity ($cm\,s^{-1}$)	Cumulative mass fraction
200	0.0480	0.0884
500	0.2416	0.2004
800	0.2957	0.3053
1000	0.3910	0.3730
1200	0.6679	0.4393
1400	1.227	0.5046
1600	2.095	0.5689
1800	2.992	0.6324
3000	7.713	1.0000

7. Results and discussion

7.1. Pulped paper analyses

Pulping rates during the tests ranged from 78 to 127 $kg\,h^{-1}$ for the high rate, and between 40 and 45 $kg\,h^{-1}$ (nominal rates were 109 and 45 $kg\,h^{-1}$). These rates were based on ream counting but were independently confirmed to within ±11% by samples analyzed from the frigate's discharge stream. For comparison purposes, all pulped paper concentrations were normalized to the nominal rates.

The concentration of pulped paper in samples collected at the three sample depths and four separation distances are shown in Table 2. Surface concentrations decreased monotonically as a function of distance in the wake behind the frigate. Surface (1.3 m) concentrations decrease from a maximum value of 0.45 $mg\,l^{-1}$ at one ship-length back to about 0.03 $mg\,l^{-1}$ at 15 ship-lengths back. While no measurable concentration of pulped paper was found at the 4.6 m sampling depth at 1 ship-length back, concentrations of about 0.03 $mg\,l^{-1}$ were observed at 10 ship-lengths. The concentration at this depth then dropped off to about 0.02 $mg\,l^{-1}$ at 15 ship-lengths. Similarly, samples collected at 8.8 m showed no measurable pulped paper concentration at 1 ship-length back, though concentrations of about 0.01 $mg\,l^{-1}$ were found at the other three downstream distances.

Table 2
Average pulped paper concentrations as a function of distance behind Frigate

Distance back (ship-lengths)	Distance back (m)	Pulped paper concentration ($mg\,l^{-1}$)			
		N	1.3 m	4.6 m	8.8 m
Measured fluorometrically					
1	126	5	0.45 ± 0.02	0.002 ± 0.003	0
5	630	12	0.10 ± 0.007	0.04 ± 0.02	0.009 ± 0.01
10	1260	13	0.03 ± 0.01	0.03 ± 0.01	0.008 ± 0.005
15	1890	7	0.03 ± 0.02	0.02 ± 0.009	0.01 ± 0.008
Measured gravimetrically					
1	126	5	0.20 ± 0.06	0	0
5	630	9	0.08 ± 0.02	0.07 ± 0.02	0.02 ± 0.01
10	1260	10	0.04 ± 0.02	0.04 ± 0.01	0.03 ± 0.02
15	1890	4	0.02 ± 0.006	0.01 ± 0.002	0.008 ± 0.005

Values were derived from fluorometric analyses. Discharge concentration was 9.6 $g\,l^{-1}$.

The resultant concentration distribution can be attributed to both dilution and particle settling. As the distance behind the frigate increases, the wake volume grows resulting in a decrease in pulped paper concentrations. However, because turbulent mixing diminishes as a function of distance behind the frigate, the effect of natural particle settling results in an increased concentration of particles at depth further back in the wake. This occurs until there is nearly a uniform distribution at 15 ship-lengths back.

The concentration of pulped material measured by both analytical methods was more likely representative of an average value rather than a maximum because of the collection method employed. Samples were collected over 6 or 10-min intervals while attempting to transit through the center of the wake. Though dye was used to assist visual tracking of the wake center, qualitative monitoring of the dye fluorometer data suggested that the measurements were not always made along the wake center. Thus, the dilution estimates determined from these results should also be considered as average values.

Based on a 9.6 $g\,l^{-1}$ input concentration, a maximum of 0.45 $mg\,l^{-1}$ of pulped paper measured at 1 ship-length represents a dilution of 2.1×10^4. Within 8 min of discharge, at 15 ship-lengths back, average particle concentrations were less than 0.03 $mg\,l^{-1}$ throughout the water column, representing a dilution of 3.2×10^5. The full range of average dilution values measured for the pulped paper particles in the field tests was 2.1×10^4 to 1.2×10^6 (Table 3).

For the most part, the two analytical methods produced comparable numbers, typically within 25%. However, differences as much as a factor of 2 to 3 were observed for a few sample sets. These differences are reasonable given the low concentration levels observed. Because the fluorometric data were not affected by variability in the background particulate load, this method is considered to be the more accurate measure of the concentrations. In either case, the two data sets analyzed independently showed excellent agreement in the concentration distribution with absolute values all nearly within a factor of 2.

Table 3
Average dilution levels of pulped paper based on a discharge concentration of 9.6 $g\,l^{-1}$

Distance back (ship-lengths)	Distance back (m)	Average particle dilution		
		Measured fluorometrically		
		1.3 m	4.6 m	8.8 m
1	126	2.1×10^4	4.8×10^6	–
5	630	9.9×10^4	2.5×10^5	1.1×10^6
10	1260	3.2×10^5	3.2×10^5	1.2×10^6
15	1890	3.6×10^5	4.8×10^5	9.6×10^5

Values were derived from fluorometric analyses.

The sample set collected at a lower discharge rate of 45 $kg\,h^{-1}$ was performed at a 5 ship-length distance. The normalized ratio of concentrations for these data and those collected at the higher 110 $kg\,h^{-1}$ rate should, therefore, be a factor of 2.4. The normalized ratio of the samples analyzed at the two discharge rates were 2.3, 2.8, and 2.6 for the three depths and averaged 2.6 indicating that the experiments run at the higher discharge rate scaled exceptionally well.

7.2. Model results

Model results for pulped material were generated for both maximum and average concentrations as a function of distance behind the frigate for both the 8- and 15-kts case. Concentration data for the 8-kts case based on an input rate of 110 $kg\,h^{-1}$ dropped from 0.32 $mg\,l^{-1}$ at roughly 500 m, where model results are stabilized, to 0.055 $mg\,l^{-1}$ out at 4000 m. Average concentrations for the same distances behind the frigate were 0.062–0.011 $mg\,l^{-1}$, respectively. The corresponding minimum dilution ranged from 1.2×10^4 and 7.2×10^4. For the 15-kts case, maximum concentrations dropped from 0.20

mg l^{-1} at roughly 500 m to 0.042 mg l^{-1} out at 4000 m. Average concentrations for the same distances behind the frigate and at 15 kts were 0.035–0.009 mg l^{-1}, respectively. The corresponding minimum dilution values were 2.0×10^4 and 9.6×10^4. The increased dilution rates calculated for the 15-kts data are primarily a result of the effective change in discharge rate per unit length of wake, a value that is proportional to the inverse of the ship speed (8/15).

7.3. Measurement and model comparison

A comparison of the measured and modeled pulp concentration data for the 8-kts case are shown in Fig. 1. All but one of the measurements fell along the model line for average values. The exception was the fluorometrically derived measurement made at 1 ship-length that fell more closely along the model maximum. Close agreement between model averages and measurements was expected because of the methods employed to collect the samples. The high data value measured close in, indicates that this sample may actually have captured the maximum concentration in the wake, though the gravimetric determination does not corroborate this. However, excellent agreement between model and measured results at greater distances behind the frigate suggests that the dilution values calculated from the model are reasonable for predicting concentrations throughout the majority of the wake. Given that the field results scaled well for varying discharge rates, the model is appropriate for estimating the concentration field under standard discharge rates (45 kg h^{-1}).

A comparison of the measured and modeled pulp dilution data for the 8-kts case is shown in Fig. 2. At 5 ship-lengths behind the frigate (630 m), the average dilution value measured was 9.9×10^4. By 15 ship-lengths (1890 m), the average pulp dilution value reached

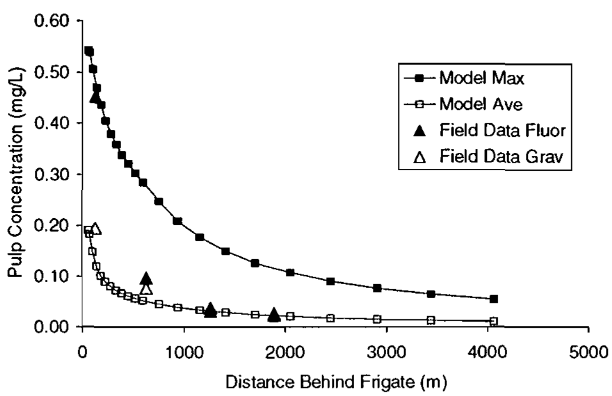

Fig. 1. Modeled and measured pulp concentrations as a function of distance behind frigate. Field data "Fluor" refers to results determined fluorometrically while "Grav" refers to results determined gravimetrically.

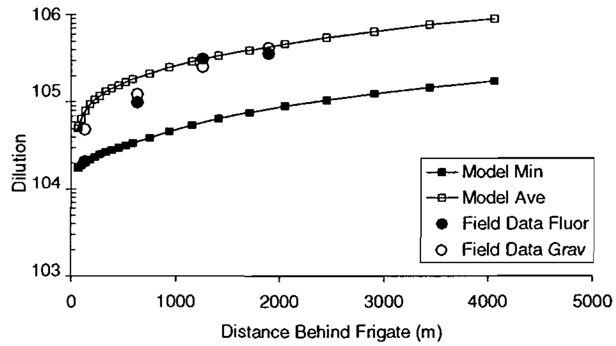

Fig. 2. Modeled and measured dilution values as a function of distance behind frigate. Field data "Fluor" refers to results determined fluorometrically while "Grav" refers to results determined gravimetrically.

3.6×10^5. Comparable model dilution values were 2.1×10^5 and 4.4×10^5, respectively. Close in the differences between the model and measurements were about a factor of 2 while out toward the end of wake mixing, the differences were within 20%. Given the standard operational discharge rate of 45 kg h^{-1}, which represents a starting discharge concentration of 3.9 g l^{-1}, the model equation predicts an average concentration of 0.018 and 0.009 mg l^{-1}, at 630 and 1890 m back. The maximum concentrations predicted at these distances, using the minimum dilution curve, would be 0.088 and 0.048 mg l^{-1}, respectively. At 8 kts, these maximum concentrations would be reached within 8 m of discharge.

7.4. Fluorescein dye analyses

Fluorescein dye was used to quantitatively measure the along-track and cross-track spreading and dilution of the liquid phase of the pulper discharge as it became entrained in the turbulent wake of the frigate. During the FTL runs dye was discharged for a continuous period of 10 min then pulsed as a way of marking the wake. Dye concentration measurements made by the flow-through fluorometers during those continuous discharges (including those during SERP runs at 1 ship-length) provide an estimate of the average dilution similar to those measured for the pulped paper. Concentrations ranged between 125 μg l^{-1} at the surface for 1 ship-length to about 9.0 μg l^{-1} at 15 ship-lengths (Table 4).

Dye measurements made during the serpentine runs provide a three-dimensional full view of the wake development as a function of time and distance behind the frigate. Examples of the dye plume for the 8-kts case are visualized as planar surface views in Fig. 3 and in vertical slices across the wake at varying distances behind the frigate in Figs. 4 and 5. It can be seen from these qualitative views that the vertical and horizontal

Table 4
Average dye concentrations and calculated dilution based on flow-through fluorometer measurements made during continuous dye discharge periods for FTL runs

Distance back (ship-lengths)	Distance back (m)	Average dye concentration ($\mu g\,l^{-1}$)		
		1.3 m	4.6 m	8.8 m
1	126	124.5	8.8	1.3
5	630	38.8	15.0	0.3
10	1260	12.8	10.3	1.5
15	1890	9.0	7.9	2.0
		Average dilution		
1	126	1.6×10^4	6.1×10^6	9.1×10^6
5	630	5.2×10^4	1.3×10^5	6.8×10^6
10	1260	1.3×10^5	2.0×10^5	1.4×10^6
15	1890	2.2×10^5	2.5×10^5	9.9×10^5

Dilution is based on a discharge concentration of $2\ g\,l^{-1}$.

boundaries of the wake generally increased with ship separation. The dye field closest to the frigate exhibited a highly concentrated region near the surface, with a concentration gradient monotonically decreasing with depth. As the wake aged, the dye spread both laterally and vertically, forming some localized patches of high dye concentrations. The wake also meandered, presumably as a result of slight variation in the frigate's track. At approximately 60 m back, the width of the wake as measured by the dye was about 15 m and reached a depth of about 5 m. The maximum width of the wake was approximately 40 m, which on this run, occurred about 2025 m downstream. At this distance back, the maximum depth of the dye was 8.5 m. Dye concentrations measured at 9.5 and 11.5 m were nearly all less than the analysis threshold value of $1\ \mu g\,l^{-1}$.

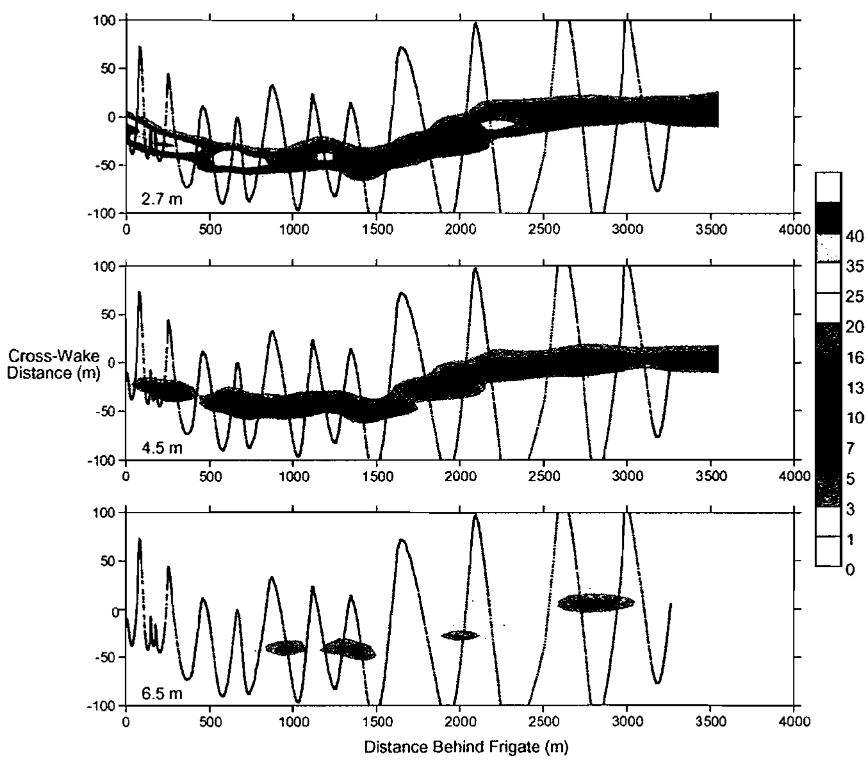

Fig. 3. Plan view dye concentration ($\mu g\,l^{-1}$) for Serpentine Run 1 (8 kts) at 2.7, 4.5, and 6.5 m depths as a function of distance behind the frigate. Concentrations were measured by the in situ fluorometer array. Dye concentrations measured at 9.5 and 11.5 m were virtually all below the $1\ \mu g\,l^{-1}$ threshold of the analysis.

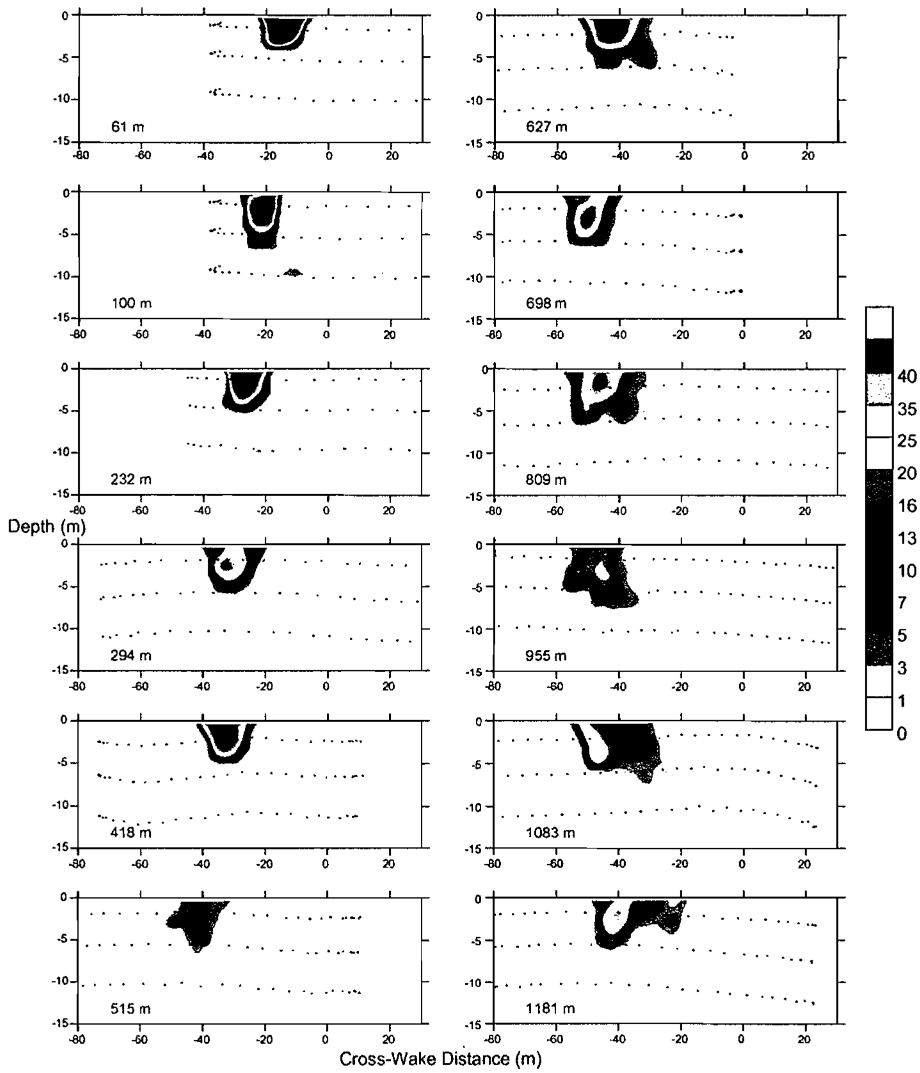

Fig. 4. Cross-sectional views (looking toward frigate from behind) of dye concentration ($\mu g\,l^{-1}$) for Serpentine Run 1 (8 kts) for first 12 full-wake crossings. The crossing distance behind the frigate is noted on each cross-section. Concentrations were measured by the in situ fluorometer array.

The dilution levels calculated for these concentrations ranged between 1.6×10^4 and 9.1×10^6. Like the pulped paper, these measurements must be considered as average values because they were not always made along the wake centerline.

Most of the dilution values based on these dye measurements were within about a factor of two of those calculated from the pulped paper measurements (see Table 4). The difference between dilution values calculated for pulped paper and those for dye increased with depth and at the shorter distances back. This resulted from the fact that while dye concentration decreases because of mixing, pulped paper concentrations diminish from a combination of both particle settling and mixing.

Results of the 15-kts tests were similar to those of the 8-kts case. As observed at 8-kts, the dye field closest to the frigate moving at 15 kts also exhibited a highly concentrated region near the surface with a concentration gradient monotonically decreasing with depth. Some patchiness and meandering were observed in these runs as well. Within about 1 ship-length, the dye had spread to a width of about 10 m and to a depth of about 5 m. The maximum width of the wake was approximately 44 m, which on this run, occurred about 3025 m downstream. At this distance back, the maximum depth of the dye was about 6 m, slightly shallower than the 8-kts case. Dye concentrations measured at 9.5 and 11.5 m were almost all less than 1 $\mu g\,l^{-1}$.

Maximum dye concentrations were almost always (~95%) found at the shallowest sensor location (2–3 m deep) and closest in to the frigate. For the 8-kts serpentine runs maximum dye concentrations measured by the in situ or flow-through fluorometers were around

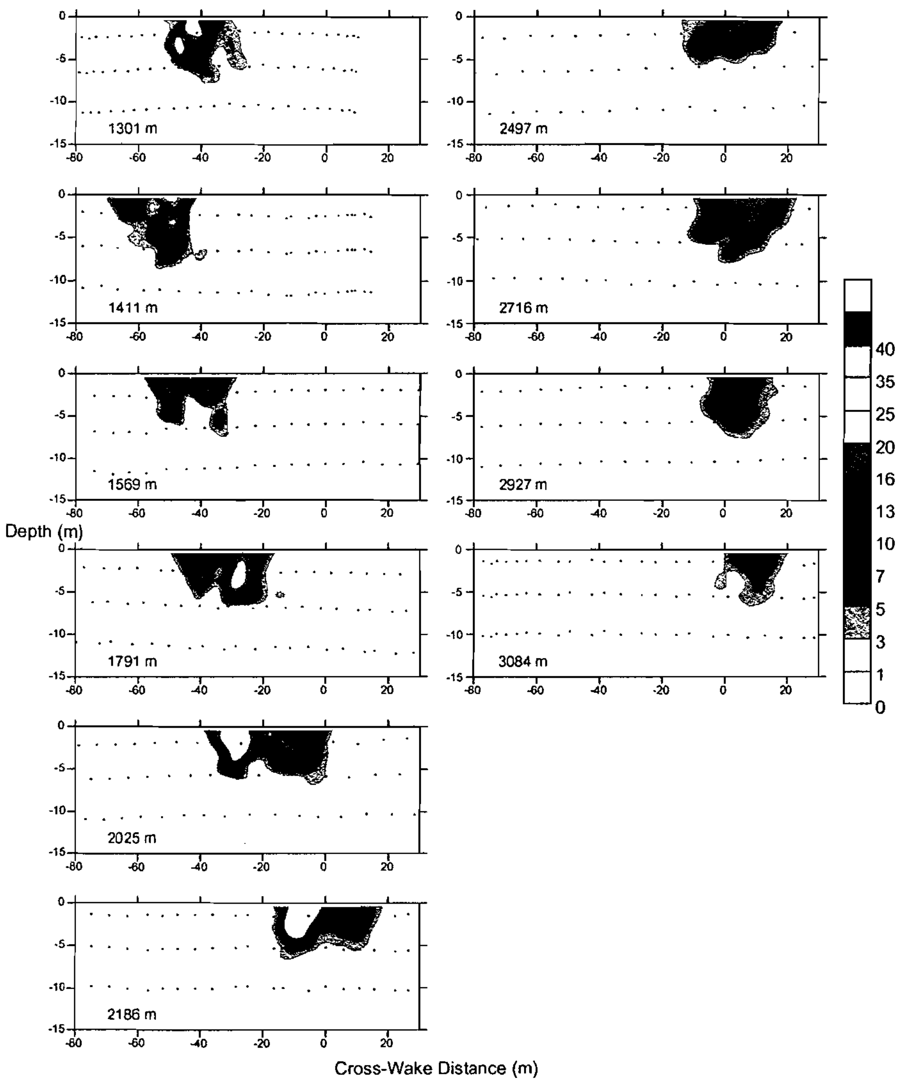

Fig. 5. Cross-sectional views (looking toward frigate) of dye concentration ($\mu g\,l^{-1}$) for Serpentine Run 1 (8 kts) for last 10 full-wake crossings. The crossing distance behind the frigate is noted on each cross-section. Concentrations were measured by the in situ fluorometer array.

350 $\mu g\,l^{-1}$, approximately 5700 times less than the discharge concentration of 2 $g\,l^{-1}$. The concentration of dye was reduced another order of magnitude within about 1500 m. At the furthest downstream measurement location of around 3500 m, concentrations were about 25 $\mu g\,l^{-1}$, reflecting a total dilution of more than 5 orders of magnitude. Close to the frigate, mean concentrations in the wake were about a factor of 20 less than the maximum, diminishing to about a factor of 3 or 4 at maximum distances back. A power law fit to the 8-kts data (Fig. 6) shows maximum dye concentration, D ($\mu g\,l^{-1}$), diminishing in the downstream direction, x (m), as: $D_{8\text{-kts}} = 9000x^{-0.77}$. This power law relationship falls roughly midway between that expected for dilution within a turbulent wake (exponent of -0.5) and dilution within a constant diffusive quiescent environment (exponent of -1) (Hyman, 1990).

At the start of the 15-kts serpentine runs, the frigate and AX were initially both moving at 8 kts (near the maximum speed of the AX). As the frigate accelerated to 15 kts, the AX began its serpentine geometry. Consequently, maximum concentration measurements obtained near the frigate for both 8- and 15-kts serpentine runs were similar (Fig. 6). However, after a few hundred meters, the speed of the frigate had increased to 15 kts and the dye concentration was, as expected, generally lower than that for the 8-kts runs. Maximum dye concentration observed during the 15-kts runs was 230 $\mu g\,l^{-1}$. In general, the ratio of maximum dye concentrations for the 15 and 8 kts runs was close to the inverse ratio of their corresponding speeds (8/15), the reasoning of which was discussed previously. Like the 8-kts case, mean concentrations in the wake were about a factor of 20 less than the maximum in close to the frigate and

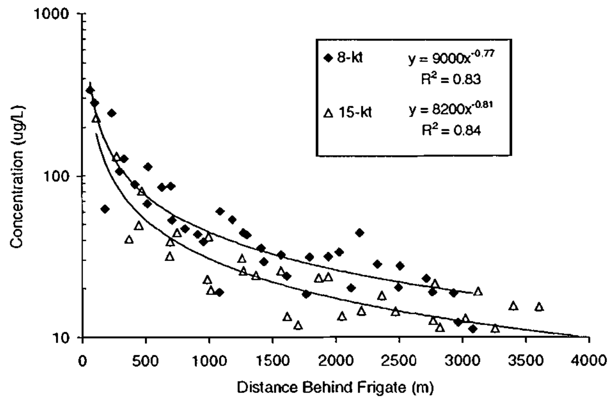

Fig. 6. Maximum dye concentration as a function of distance behind the frigate for both 8- and 15-kts runs measured at each wake crossing.

diminished to about a factor of 3 or 4 at maximum distances back. A power law fit to the 15-kts at-sea data show maximum dye concentrations diminishing in the downstream direction as: $D_{8\text{-kts}} = 8200x^{-0.81}$.

Minimum dye dilution estimates were made by dividing the concentration of dye at discharge (2 $\mathrm{g\,l^{-1}}$), by the maximum concentration measured at each wake crossing by the in situ fluorometer array. Fig. 7 displays the minimum dye dilution as a function of frigate–*AX* separation distance for both frigate speeds of 8 and 15 kts. Minimum dilution ranged from about 5.9×10^3 closest in to the frigate to roughly 2.3×10^5 at a downwake distance of 3500 m for both the 8- and 15-kts case. The 15-kts dilution values were almost always lower than the 8-kts counterpart, though the differences were very slight. Data derived from the in situ fluorometer array are corroborated with the flow-through dye measurements as shown. As anticipated, close to the frigate, the flow-through fluorometers measured noticeably higher dilution than the in situ system because of the

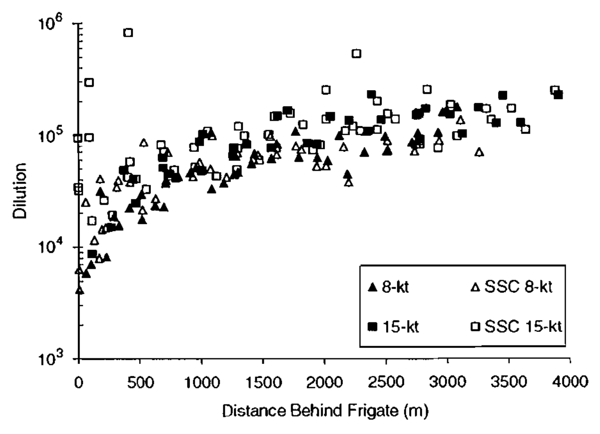

Fig. 7. Minimum dye dilution for 8- and 15-kts data based on 2 $\mathrm{g\,l^{-1}}$ discharge concentration. Results for data collected from both the in situ fluorometer array and flow-through array (SSC) are included.

additional mixing occurring in the sampling tubes. However, with increasing distance from the frigate, this difference generally decreased because the additional mixing in the flow-through tube became less important as the gradient in the wake was reduced.

7.5. Model results

Results of the model simulation for dye are shown qualitatively in Figs. 8 and 9. A plan view of the 8-kts data shows growth of the wake from the IDP at about 60 m out to 4000 m for 2.5 m depth. The plot uses the same contour colors and intervals and minimum threshold of 1 $\mathrm{\mu g\,l^{-1}}$ as shown earlier for the field data. Though data are shown starting at the IDP, only values for distances greater than 500 m are considered fully stabilized. Unlike the measured data that showed patchiness and a meandering track, the concentration contours for the model output were smooth and straight throughout. Cross-sectional plots show the wake growing monotonically both laterally and with depth. Growth of the wake to the right of centerline in the plot results from the model taking into consideration the starboard discharge and clockwise rotation of the screw. The modeled wake width grew from its preset value of about 10 m at the IDP to about 30 m out at 3000 m. Over these distances, the depth of the dye increased from about 3–10 m.

Maximum concentrations of dye decreased from 66 $\mathrm{\mu g\,l^{-1}}$ at 500 m behind the frigate to 14 $\mathrm{\mu g\,l^{-1}}$ out at 4000 m. A power law fit to the modeled 8-kts data show maximum dye concentrations diminishing in the downstream direction as: $D_{8\text{-kts}} = 5778x^{-0.73}$, where x is measured in meters. Again, this relationship is midway between fully turbulent (−0.5 exponent) and diffusive dilution (−1 exponent), Hyman, 1990. Mean dye concentrations in the wake at each crossing were about a factor of 4 to 5 less than maximum values throughout the wake. Minimum downstream dilution, calculated by dividing the concentration of effluent at discharge (2 $\mathrm{g\,l^{-1}}$) by the maximum concentration of effluent computed in the corresponding wake cross section, ranged from 3.0×10^4 to 1.4×10^5 over the 500–4000 m downwake distance.

Corresponding model results for the 15-kts case showed a decrease in maximum dye concentration initially 42 $\mathrm{\mu g\,l^{-1}}$ at 500 m behind the frigate to 8.6 $\mathrm{\mu g\,l^{-1}}$ at 4000 m. These values correspond to minimum dilution levels of 4.8×10^4 and 2.3×10^5. A power law fit to the modeled 8-kts data show maximum dye concentrations diminishing in the downstream direction as: $D_{15\text{-kts}} = 6410x^{-0.80}$.

Because the governing equations for the model were originally ensemble averaged, the maximum concentrations calculated are most likely less than those found in any single realization. It has been estimated for bubble

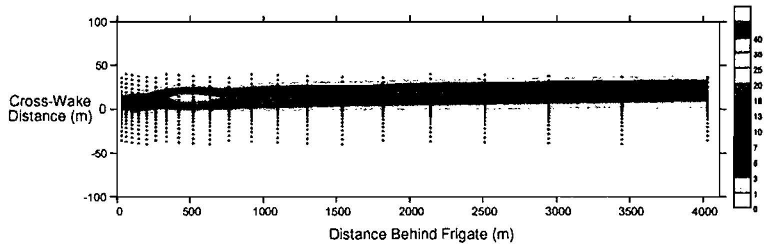

Fig. 8. Plan view of modeled dye concentration ($\mu g\, l^{-1}$) at 2.5 m depth, as a function of distance behind the frigate at 8 kts. The IDP was at about 60 m for this run.

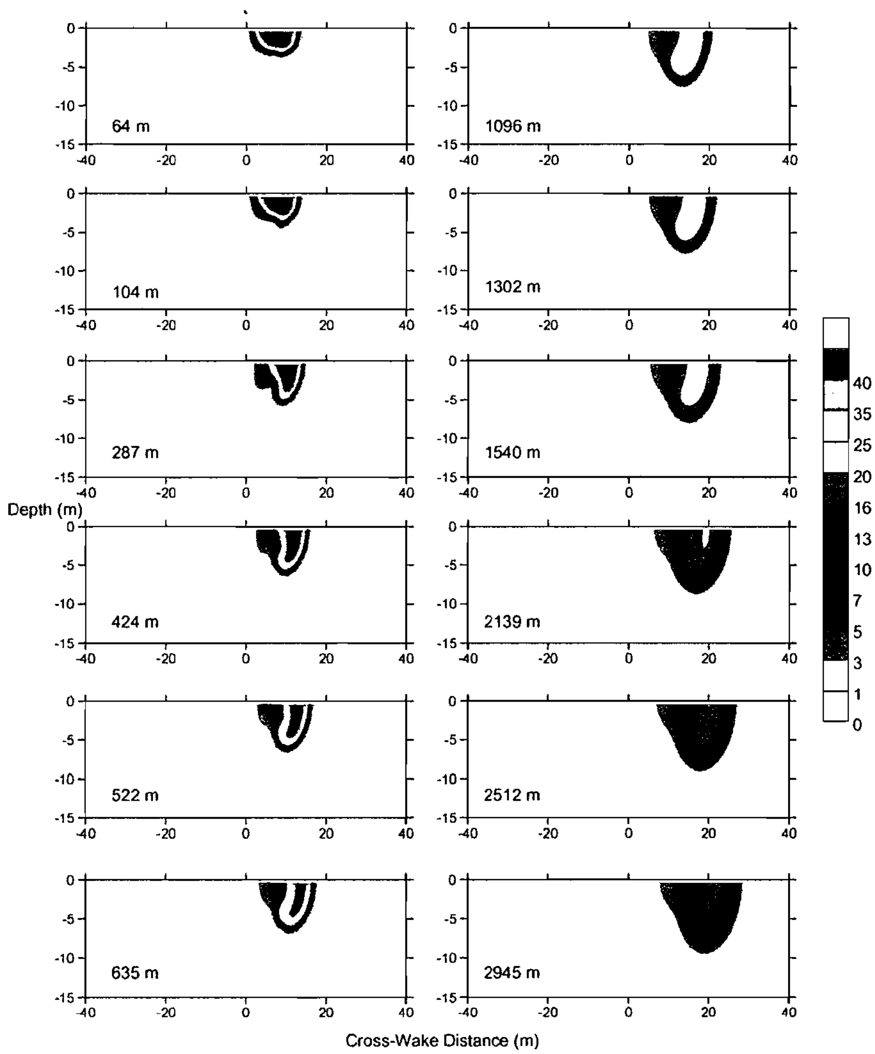

Fig. 9. Cross-sectional views (looking toward frigate from behind) of model dye concentration ($\mu g\, l^{-1}$) at selected distances behind the frigate for the 8-kts case. The IDP was at about 60 m for this run.

concentrations (original model output) that the difference between maximum concentrations found for single realizations and ensemble averages are typically less than 50%. It is expected that a roughly similar relationship should hold for the present model estimates.

7.6. Measurement and model comparison

The field measurements and model runs of dye discharge provide a means to assess the dilution of the liquid phase of the discharge stream as well as to provide details of wake dilution not available from the spatially sparse particle data set. These data, in terms of dilution versus distance behind the frigate, are shown in Fig. 10. For the 8-kts case, it can be seen that the measured and modeled data agree remarkably well with each other. This is true for both the in situ and flow-through field measurements. A regression equation using the in situ data predicts minimum dilution values of 3.1×10^4 and 7.3×10^4 at 5 and 15 ship-lengths back, respectively. For the same locations, the model equation predicts minimum dilution values of 3.9×10^4 and 8.7×10^4, respectively. Model predictions of average dilution values are roughly a factor of 4 higher at 1.7×10^5 and 3.4×10^5. The measured values and model predictions are typically within 20% of one another, though at 5 ship-lengths, the average dilution values differ from the model by 50%. The average dye dilution value of 3.4×10^5 predicted by the model near the end of wake mixing is comparable to the value of 3.6×10^5 calculated from measurements.

Similarly good agreement was observed for the 15-kts case. A regression equation using the in situ data predicts minimum dilution values of 2.8×10^5 and 4.5×10^5 at 5 and 15 ship-lengths back, respectively. Using the model equation, the minimum dilution at 5 ship-lengths is 3.5×10^4, while at 15 ship-lengths back it is 6.3×10^5. The difference between model and measured values was about 20%. Average dilution values are about a factor of 5 higher for corresponding distances, at 5.5×10^4 and 1.3×10^5.

The exceptional agreement between field measurements made using multiple independent techniques for both the liquid phase (dye) and particulate phase (pulp) of the discharge suggests that wake dilution was very well characterized by the field effort. The remarkable

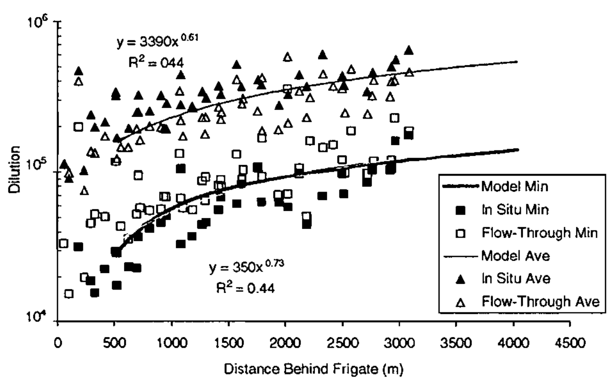

Fig. 10. Modeled and measured minimum and average dilution values as a function of distance behind frigate for the 8-kts case. The equations shown are for the modeled data.

agreement of the field data with model prediction supports a strong reliance on the model for use in predictions for conditions falling outside the specific measurement regime.

8. Results validation

Excellent agreement between field measurements and model results suggest that the model can be used to accurately predict the near-field space–time distribution of shipboard discharges. How well the results compare to physical laws and governing theory indicates how well they can be used universally rather than empirically. Physical laws governing wake growth and conservation of mass provide confidence in how well the field data sampled the wake (Okubo, 1971; Csanady, 1981) and how transportable these results are to other conditions.

8.1. Wake width

Dye measurements made during the serpentine runs were used in validating physical laws governing wake growth. The dye stream width at each downstream location was determined as the greatest lateral extent of dye throughout the measurement plane for concentrations greater than or equal to 1 $\mu g\, l^{-1}$. The width, W (m) for both ship speeds increased with downstream separation distance x (m) from the frigate approximately as: $W \sim x^{0.25}$ at 8 kts and $W \sim x^{0.31}$ for 15 kts. Though the regression equation for the 8-kts case had a r^2 of 0.7, noise in the 15-kts case resulted in a regression equation r^2 of only 0.4. Computer simulations, which represent an average result acquired over many realizations, predict that the plume width should increase downstream as $W \sim x^{0.20}$ and $W \sim x^{0.18}$ for the 8- and 15-kts runs, respectively.

The observed power law increase of wake width with downstream distance for both the field experiments measurements and model simulations lie well within the range of exponents reported for various types of wakes studied in the laboratory. Investigations conducted in tanks with unstratified flows have reported growth rates for the width of submerged, momentumless wakes, ranging from $W \sim x^{0.25}$ (Naudascher, 1965; Meritt, 1972) to $W \sim x^{0.33}$ (Schooley and Stewart, 1963). Meritt (1972) studied the effect of stratification on wake growth in the laboratory and found that during the period when the wake is collapsing vertically, horizontal wake growth is enhanced with $W \sim x^{0.5}$. While the present experiments were made with a mixed-layer depth of 20 m significantly larger than the wake depth, inhibited vertical motion at the air–sea interface is expected to increase the horizontal growth rate. The excellent agreement between measured wake growth in the field and that measured in laboratory studies lends a high degree of confidence that the dye plume was well characterized and follows physical relationships determined under controlled conditions in the laboratory.

8.2. Mass balance

The dye discharge rate for both frigate speeds was 6.7 $g\,s^{-1}$. At 8 kts, the mass discharged per meter length was, therefore, 1.67 g, while at 15 kts, the mass discharge was 0.89 $g\,m^{-1}$. To look at how well mass was conserved, the mass of dye at each crossing was calculated by integrating data over the whole cross-section using all measurements above the 1 $\mu g\,l^{-1}$ threshold. While there was considerable variability from section to section, average values calculated for 500 m and further out were in good agreement with the expected values. For the 8-kts case the average was 1.89 ± 0.91 $g\,m^{-1}$, approximately 13% higher than expected. For the 15-kts case, the average mass calculated was 0.90 ± 0.37 $g\,m^{-1}$, or roughly 1% higher than expected. The good agreement between average measured and predicted amounts suggests that the dye field was effectively sampled.

The mass balance of pulped paper was also assessed assuming that pulped particles did not appreciably settle out below the sample intakes during the FTL runs. Based on a discharge rate of 30.5 $g\,s^{-1}$ pulped paper and 6.7 $g\,s^{-1}$ dye, the ratio of their masses was fixed at a value of 4.6. The ratio of particles to dye was then calculated at each fixed distance back and at all depths, then compared to this discharge ratio. Ratio values derived from the fluorometric data were within 25% of the expected value for each cross-section while the ratio determined from the gravimetric data varied by nearly a factor of 2. The largest differences were observed at 8.8 m where the mass of both particles and dye were lowest. The average ratio overall was 4.5 (4.1 fluorometric, 5.6 gravimetric) in excellent agreement with the expected value. These results suggest that mass was sufficiently conserved during the FTL runs and that the measurements provided a sufficient characterization of the wake.

9. Conclusions

Full-scale field tests successfully characterized wake dilution of the effluent discharged from the frigate, USS *Vandergrift*. Independent measurements of pulped paper and of dye made simultaneously at multiple depths and at variable distances behind the frigate were sufficient to define the average and minimum dilution levels throughout the wake. By the end of wake mixing for the 8-kts case, or at about 15 ship-lengths, measured pulp dilution values reached 3.6×10^5 while comparable model dilution values were 4.4×10^5, a difference of about 20%. These values represent average dilution levels in the wake that would be present within 8 minutes after discharge. The comparable average dilution measured and modeled for the liquid phase of the discharge is 3.9×10^5 and 3.4×10^5, respectively, a difference of about 15%. Measured and modeled minimum dilution levels for these same conditions would be 7.3×10^4 and 8.7×10^4, respectively. The dilution levels for a frigate traveling at 15 kts were about double the 8-kts case.

The full-scale field tests were also successful in validating TBWAKE model predictions. Though differences of nearly a factor of 2 were found between model predictions and pulp concentrations measured within 5 ship-lengths of the frigate, the differences were typically less than 20% at the end of wake mixing. The excellent agreement between model and field results suggests that TBWAKE can be used successfully to predict the short-term fate of solid wastes discharged from ships under a variety of conditions. The concentration estimates provided by the model were critical in determining the potential ecological impacts of solid waste discharges from US Navy ships.

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